Math 115

Summer 2017

Lecture 4

$\sqrt{xy}$

Solving Linear Inequalities

final ans will be

$x > a$ $\qquad\qquad x \geq a$

$x < a$ $\qquad\qquad x \leq a$

$a < x < b$ $\qquad\qquad b < x \leq a$

$\phantom{a < x < b}\qquad\qquad b \leq x < a$

we perform all steps $\qquad b \leq x \leq a$

as we did with Solving

linear equations, however, we must reverse
inequality whenever we divide, or Multiply by a
Negative #.

Solve

$2x - 3 < 7$

$2x < 7 + 3$

$2x < 10$

$\frac{2}{2}x < \frac{10}{2}$

$\boxed{x < 5}$

Solve

$-3x + 4 > 19$

$-3x > 19 - 4$

$-3x \boxed{>} 15$

$\frac{-3}{-3}x < \frac{15}{-3}$

$\boxed{x < -5}$

There are 3 ways to express final ans.
- Set-Builder notation
- Interval notation
- Graphing

Solve

$2(x - 8) - 6x \le 16$

$2x - 16 - 6x \le 16$

$-4x - 16 \le 16$

$-4x \le 16 + 16$

$-4x \boxed{\le} 32$

$\frac{-4}{-4}x \ge \frac{32}{-4}$

$\boxed{x \ge -8}$

Solve

$\frac{1}{2}x + \frac{5}{6} \ge \frac{3}{4}x - \frac{5}{12}$

Use LCD to Clear fractions

LCD = 12

$\overset{6}{\cancel{12}} \cdot \frac{1}{2}x + \overset{2}{\cancel{12}} \cdot \frac{5}{6} \ge \overset{3}{\cancel{12}} \cdot \frac{3}{4}x - \cancel{12} \cdot \frac{5}{\cancel{12}}$

$6x + 10 \ge 9x - 5$

$6x - 9x \ge -5 - 10$

$-3x \boxed{\ge} -15$

$\frac{-3}{-3}x \le \frac{-15}{-3}$
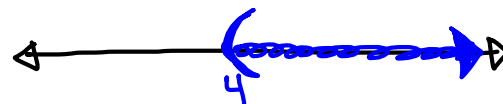
$\boxed{x \le 5}$

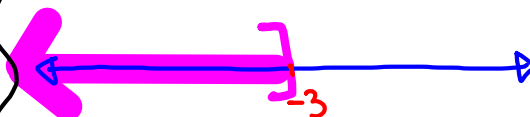Set-Builder notation:

$\{ x \mid \boxed{x \leq 5} \}$

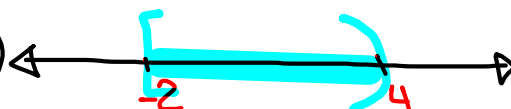↗ Such that

$\{ x \mid x > -4 \}$

$\{ x \mid -7 \leq x < 3 \}$

Graphing

$x > 4$

$x \leq -3$

$-2 \leq x < 4$

Interval notation

-2   ∞

$[-2, \infty)$

$-\infty$ 3

$(-\infty, 3)$

-4   7
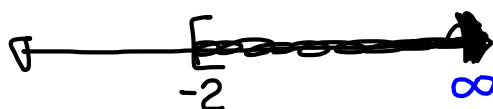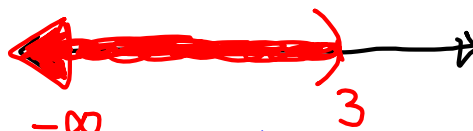
$[-4, 7]$

use [or]
with ≥ or ≤
use (or)
with > or <

Solve, express final ans in all 3 ways.

$-4x - 8 > 12$

$-4x > 12 + 8$

$-4x \boxed{>} 20$

$\dfrac{-4}{-4}x < \dfrac{20}{-4}$

$\boxed{x < -5}$

① Set-Builder notation

$$\{x \mid x < -5\}$$

② Graph

$-\infty$ -5
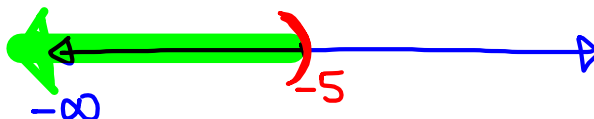
③ Interval notation

$$(-\infty, -5)$$

Solve, express final ans in all 3 ways

$-2x + 8 \leq 3x - 22$
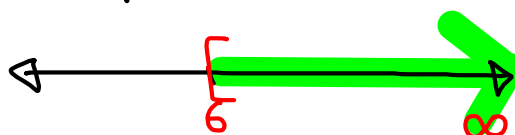
$-2x - 3x \leq -22 - 8$

$-5x \boxed{\leq} -30$

$\dfrac{-5}{-5}x \geq \dfrac{-30}{-5}$

$x \geq 6$

① S.B.N.

$$\{x \mid x \geq 6\}$$

② Graph

6 ∞

③ I.N. $[6, \infty)$

$$\frac{2}{5}(x-3) < \frac{3}{4}(x+2) + 1$$

Solve, express final answer in all 3 ways.

Use LCD = 20 to clear fractions.

$$20 \cdot \frac{2}{5}(x-3) < 20 \cdot \frac{3}{4}(x+2) + 20 \cdot 1$$

$$8(x-3) < 15(x+2) + 20$$

$$8x - 24 < 15x + 30 + 20$$

$$8x - 15x < 50 + 24$$

$$-7x < 74$$

$$\frac{-7}{-7}x > \frac{74}{-7}$$

$$x > -\frac{74}{7}$$

① S.B.N.

$$\left\{ x \mid x > \frac{-74}{7} \right\}$$

② Graph

③ I.N. $\left(-\frac{74}{7}, \infty\right)$

$-\frac{74}{7}$ ∞

Solve $\quad -3 < 2x + 5 \le 13$

we want to isolate the variable in the middle.

$$-3 - 5 < 2x + 5 - 5 \le 13 - 5$$

$$-8 < 2x \le 8$$

$$\frac{-8}{2} < \frac{2}{2}x \le \frac{8}{2}$$

$$-4 < x \le 4$$

① S.B.N.

$$\left\{ x \mid -4 < x \le 4 \right\}$$

② Graphing

$-4 \qquad 4$

③ I.N.

$$(-4, 4]$$

Solve $5 \leq 3x - 4 < 17$

Add 4 $5 + 4 \leq 3x - 4 + 4 < 17 + 4$
to undo
Subtraction $9 \leq 3x < 21$

Divide by 3 $\dfrac{9}{3} \leq \dfrac{3}{3}x < \dfrac{21}{3}$

to undo Multiplication

$3 \leq x < 7$

① S.B.N.

$\{x \mid 3 \leq x < 7\}$

② Graph

3   7

③ I.N. $[3, 7)$

Solve, Give Ans in all 3 ways:

$2 < -3x + 2 \leq 17$

$2 - 2 < -3x \leq 17 - 2$

$0 < -3x \leq 15$
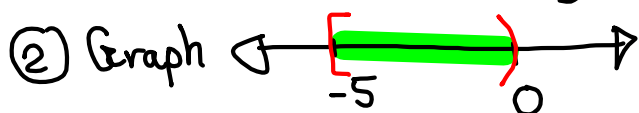
Divide by -3

$\dfrac{0}{-3} > \dfrac{-3}{-3}x > \dfrac{15}{-3}$

$0 > x \geq -5$

It is better and less confusing to have smaller # of the left side.

$-5 \leq x < 0$

① S.B.N. $\{x \mid -5 \leq x < 0\}$

② Graph

-5   0

③ I.N. $[-5, 0)$

The sum of two numbers is 67. One of them is 1 less than 3 times the other one. find both numbers.

First # $\Rightarrow x$

Second # $\Rightarrow 3x-1$

The numbers are 17 & 50.

First + Second = 67

$x + 3x-1 = 67$

$4x = 68$

$x = 17$

PTA purchased 42 Tkts for a trip to the zoo.

The number of kids was 2 fewer than 3 times the number of adults.

How many kids?

Kids $\longrightarrow 3x - 2$

Adults $\longrightarrow x$

$3(11)-2$

$= 33 - 2$

$= 31$

31 kids

Kids + Adults = 42

$\boxed{3x-2} + \boxed{x} = 42$

$4x - 2 = 42$

$4x = 44$

$x = 11$

Lisa ordered 76 color pens.

Red, Blue, Green.

# of blue pens was twice # of red pens.

# of green pens was 8 less than 4 times the number of red pens. How many of each?

Red → x

Blue → $2x$

Green → $4x-8$

$$x + 2x + 4x - 8 = 76$$
$$7x = 84 \qquad \boxed{x = 12}$$

12 Red, 24 Blue, and 40 Green Pens.

John had a piece of wood $\underline{33 \text{ ft}}$ long.

He cut it into 3 pieces.

Second piece was half of the first piece.

Third piece was one-third of the first piece.

18 ft, 9 ft, and 6 ft

find all three pieces.

First → x

Second → $\frac{1}{2}x$

Third → $\frac{1}{3}x$

$$\boxed{x} + \boxed{\tfrac{1}{2}x} + \boxed{\tfrac{1}{3}x} = 33$$

$$LCD = 6$$
$$6x + 3x + 2x = 6 \cdot 33$$
$$11x = 6 \cdot 33 \qquad \boxed{x = 18}$$

WP 4 → Due Monday

Solve & identify the type of equation:

$$2(3x - 5) - 4(x + 8) = 2(x + 3) - 48$$

$$6x \boxed{-10} - 4x \boxed{-32} = 2x + 6 - 48$$

$$2x - 42 = 2x - 42$$

$$2x - 2x = -42 + 42$$

$$0 = 0 \quad \text{infinitely many solns.}$$

$$\text{Identity.}$$

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1) Solve $z = \dfrac{x - M}{S}$ for x.   Hint: Clear Fraction

$$LCD = S$$

$$Sz = x - M$$

$$\boxed{Sz + M = x}$$

② Solve $A = \dfrac{h(B + b)}{2}$ for h.

$$2A = h(B + b)$$

$$\frac{2A}{B + b} = h$$

③ Solve $A = \dfrac{h(B+b)}{2}$ for B.

$$2A = h(B+b)$$

$$2A = hB + hb$$

$$2A - hb = hB$$

$$\boxed{\dfrac{2A - hb}{h} = B}$$

Solve for y, write ans in the form of

$$y = mx + b$$

$(2x) - 3y = 9$

$-3y = -2x + 9$

$y = \dfrac{-2}{-3}x + \dfrac{9}{-3}$

$\boxed{y = \dfrac{2}{3}x - 3}$

$4x + 2y = 10$

Divisible by 2

$(2x) + y = 5$

$\boxed{y = -2x + 5}$

Solve for y

$2x + 3y \leq 6$

$3y \leq -2x + 6$

$y \leq \dfrac{-2}{3}x + \dfrac{6}{3}$

$\boxed{y \leq \dfrac{-2}{3}x + 2}$

$\left\{ \underparen{3x} - 4y > 12 \right.$

$-4y \boxed{>} -3x + 12$

Divide by -4

$y < \dfrac{-3}{-4}x + \dfrac{12}{-4}$

$\boxed{y < \dfrac{3}{4}x - 3}$

Simplify

$\dfrac{3}{5}\left(10x^2 - 15x + 5\right) - 6x^2 + 9x - 3$

$= \dfrac{3}{5} \cdot \overset{2}{\cancel{10}}x^2 - \dfrac{3}{5} \cdot \overset{3}{\cancel{15}}x + \dfrac{3}{\cancel{5}} \cdot \cancel{5} - 6x^2 + 9x - 3$

$= 6x^2 - 9x + 3 - 6x^2 + 9x - 3$

$= \boxed{0}$
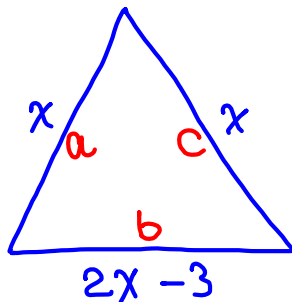
Due Monday:
- WP 4
- WP 5
- SG 3, 4 & 5

CQ3 ⇒ Due Sunday

CQ2 ⇒ Friday

Two sides of a triangle are equal. The third side is 3 cm shorter than the sum of equal sides.

1) Draw & label

2) Find all three sides if the perimeter is 25cm.

$P = 25$

$$\boxed{a} + \boxed{b} + \boxed{c} = 25$$
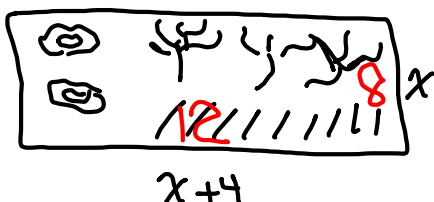
$$\boxed{x} + \boxed{2x-3} + \boxed{x} = 25$$

$$\vdots$$

$$x = 7$$

7cm, 7cm, and 11cm.

Triangle labeled: sides x, x, base $2x - 3$; angles a, c, b.

A rectangular garden has a perimeter of 40 m.

Its length is 4m longer than its width.

1) Draw & label

2) Find its dimensions

$$P = 40$$
$$2L + 2W = 40$$
$$2(x+4) + 2x = 40$$
$$\vdots$$
$$x = 8$$

$x+4$

x

8 m by 12m.

A triangular Carpet has a perimeter of 52 ft.

One Side is twice another side.

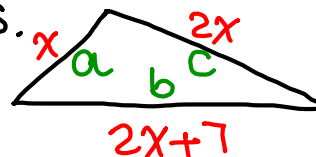Third Side is 7 ft more than the longer Side of the first two Sides.

Find the third Side.

$2x$
x a b c
$2x+7$

$$P = 52$$
$$a + b + c = 52$$
$$x + 2x + 7 + 2x = 52$$
$$5x = 52 - 7$$
$$5x = 45$$
$$x = 9$$

$$2(9) + 7$$
$$= 25$$

Third Side is 25 ft.

A rectangular classroom has a perimeter of 144 ft.

The length is 2 ft shorter than 3 times its width.
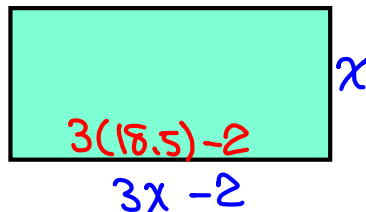
find its dimensions.

$P = 144$

$2L + 2W = 144$

$2(3x-2) + 2x = 144$

$\vdots$

$x = 18.5$

$3(18.5) - 2$

$3x - 2$

x

18.5 ft by 53.5 ft

Lisa got 72 and 88 on first 2 exams. In order to get B for the class, she needs an average of at least 80.

Final exam counts as 2 tests.

what Score on the final exam does she need to get a B?

Average ≥ 80

↑

at least

$\dfrac{\text{Total Pts}}{\text{\# of exams}} \geq 80$

$\dfrac{72 + 88 + 2F}{4} \geq 80$

Solve $\dfrac{72 + 88 + 2F}{4} \geq 80$

LCD = 4

$72 + 88 + 2F \geq 4(80)$

$160 + 2F \geq 320$

$2F \geq 320 - 160$

$2F \geq 160$ $\qquad$ $F \geq 80$

Lisa needs at least 80 on the final to secure a B grade.

Jessica is moving to a new location. She has $100 budget. Rental truck goes for $25/day & 50¢ per mile. Find the distance that she can move to.

Total Cost at most $100

$M \leq \dfrac{75}{.5}$

$M \leq 150$

75 miles radius

Cost ≤ 100

$25 + \boxed{.50}M \leq 100$

$.5M \leq 100 - 25$

$.5M \leq 75$

Jose has a small business.

He needs to open a checking account.

B of A $\Rightarrow$ $5/Month + 8¢/check

wells fargo $\Rightarrow$ $10/Month + 3¢/check

find the number of checks in any month
that he can write and wells fargo would be
a better deal.

Cost less

Cost Wells Fargo $<$ Cost B of A

$10 + .03C < 5 + .08C$

$.03C - .08C < 5 - 10$

$-.05C < -5$

$C > \dfrac{-5}{-.05}$ $C > 100$

Wells Fargo is a better deal
if Jose writes more than 100 Checks
Per month.